Counting problems in groups with contracting elements

杨文元
北京大学

In this talk, we shall study a class of groups with contracting elements and survey several counting results in this class of groups. We will explain a basic tool called extension lemma in obtaining those counting results. This tool could be thought of as an orbit closing lemma in group theory.

Every closed surface of genus at least 18 is Loewner

苏伟旭
中山大学

We obtain an improved upper bound involving the systole and area for the volume entropy of a Riemannian surface. As a result, we show that every orientable and closed Riemannian surface of genus at least 18 satisfies Loewner's systolic ratio inequality. We also show that every closed orientable and nonpositively curved Riemannian surface of genus at least 11 satisfies Loewner's systolic ratio inequality. This is a joint work with Qiongling Li.

Averages of determinants of Laplacians over moduli spaces for large genus

贺宇昕
清华大学

Let $\mathcal{M}_g$ be the moduli space of hyperbolic surfaces of genus g endowed with the Weil-Petersson metric. We view the regularized determinant $\log \det(\Delta_X)$ of Laplacian as a function on $\mathcal{M}_g$ and show that there exists a universal constant $E > 0$ such that as $g \to \infty$,

(1) the expected value of $\left| \frac{\log \det(\Delta_X)}{4\pi(g-1)} - E \right|$ over $\mathcal{M}_g$ has rate of decay $g^{-\delta}$ for some uniform constant $\delta \in (0, 1)$;

(2) the expected value of $\left| \frac{\log \det(\Delta_X)}{4\pi(g-1)} \right|^{\beta}$ over $\mathcal{M}_g$ approaches to E^{β} whenever $\beta \in [1, 2)$.

This is a joint work with professor Yunhui Wu.

On the mapping class groups of 4-manifolds with 1-handles

谢羿
北京大学

Budney and Gabai proved that the mapping class group of the product of a circle and a 3-dimensional disk is an abelian group of infinite rank. Their proof relies on an invariant defined using the configuration spaces of ordered points. In this talk, we will generalize Budney-Gabai's invariant and use it to demonstrate that the smooth and topological mapping class groups of certain 4-manifolds with 1-handles contain infinite rank abelian subgroups. This is joint work with Jianfeng Lin and

Boyu Zhang.

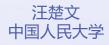## Periodic Phenomena in the C-Motivic Sphere Spectrum

李谷川
北京大学

Through the J-homomorphism, Bott periodicity reveals a periodic structure in the stable homotopy groups of spheres, offering key insights into problems such as vector fields on spheres. In the motivic setting, the Hopf map η is non-nilpotent and gives rise to the first exotic periodicity, which has been studied by Bachmann and Hopkins. In this talk, we present a study of the second exotic periodic phenomenon in the C-motivic sphere. This is joint work with Dan Isaksen, Hana Jia Kong, Yangyang Ruan, and Heyi Zhu.

Marked length spectrum rigidity in groups with contracting elements

万仁星
华东师范大学

The marked length spectrum (MLS) of a closed negatively curved Riemannian manifold M is a function defined on $\pi_1(M)$ that maps each conjugacy class to the length of its geodesic representative. The well-known marked length spectrum rigidity conjecture states that if two negatively curved Riemannian metrics on a closed manifold have the same marked length spectrum, then they are isometric. In this talk, we introduce a coarse-geometric version of MLS rigidity problem in GGT and give an affirmative answer to groups with contracting elements. This is joint work with Xiaoyu Xu and Wenyuan Yang.

Recent advances in the Volume Conjecture

汪楚文
中国人民大学

I will introduce different versions of the volume conjecture, including the Kashaev-Murakami-Murakami volume conjecture, the Chen-Yang volume conjecture, and the SU(n) volume conjecture by Chen-Liu-Zhu. I will also present recent progress on these volume conjectures. Some techniques for proving the volume conjectures will be discussed as well.

Combinatorial Ricci flows on infinite disk triangulations

周朴淳
复旦大学

In this talk, we introduce combinatorial Ricci flows (CRF) in Euclidean and hyperbolic background geometries on infinite disk triangulations, which are discrete analogs of Ricci flows on simply connected open surfaces. We establish the well-posedness results of CRF in both Euclidean and hyperbolic background geometries. We further prove two convergence results of CRF for both

recurrent and transient triangulations, which indicate a uniformization theorem for CRF on infinite disk triangulations. As an application, we prove an existence result of circle-packing metrics with infinite prescribed cone angles in hyperbolic background geometry. This report is from the joint work of Huabin Ge and Bobo Hua.

Z/2 Harmonic 1-Forms and 3-Manifold Topology

陈家煌
中国科学院

The concept of Z/2 harmonic 1-forms, introduced by Clifford Taubes, plays a key role in generalizing the Uhlenbeck compactness theorem. These 1-forms can be regarded as the 3-dimensional analog of quadratic differentials, and can be used to compactify the SL(2, C) character variety . In this talk, we will explore intriguing connections and open questions linking Z/2 harmonic 1-forms to the topology of 3-manifolds.

Iwip endomorphisms of free groups and fixed points of graph selfmaps

王鹏
西安交通大学

In a paper from 2011, Jiang, Wang and Zhang studied the fixed points and fixed subgroups of graph selfmaps on a connected finite graph or a connected compact hyperbolic surface X. In particular, they proved that a certain quantity defined in terms of characteristics and indices of fixed point classes of selfmaps on X is bounded below by $2\chi(X)$, where $\chi(X)$ is the Euler characteristic of X. In this talk, we give a sufficient condition for when equality holds and we partially answer a question of Jiang. We do this by studying iwip outer endomorphisms of free groups acting on stable trees. This is a joint work with Qiang Zhang

Thurston measure of unit balls associated to quadratic differentials

张慎行
复旦大学

In 2008, Mirzakhani introduced Mirzakhani function to study the counting problem of simple closed curves on hyperbolic surfaces, and proved its properness and integrability. Precisely, the function is defined by the Thurston measure of the unit ball associated to hyperbolic length function in the measured lamination space.
Similarly, a quadratic differential defines a flat metric on a closed surface of genus ≥ 2 and a flat length function in the measured lamination space. Consider the Thurston measure of the unit ball associated to the flat length function, which becomes a function on the moduli space of quadratic differentials. We study the asymptotic behaviour of the function when a family of quadratic differentials goes to the boundary of the moduli space under the Deligne-Mumford compactification, and give an upper and lower bound of the function, which depends on the comparision between the flat structure and its hyperbolic structure. Finally, we investigate its integrability. This is a joint work with Weixu Su.

黎曼面上 A 型 Toda 系统的有限能量解

许斌
中国科学技术大学

户田盛和于 1967 年奠基的一维 Toda 系统，通过常微分方程组刻画了具有指数势的弹簧振子链的动力学规律。随着理论发展，该模型被拓展至二维情形，并逐渐显露出其与黎曼曲面到李群或对称空间的调和映射理论之间的深刻对应关系。值得说明的是，系统的核心动力学特征由对应复单李代数的 Cartan 矩阵所决定， 其中 A 型 Toda 系统尤为特殊地实现了 Liouville 方程（描述球度量）向高维情形的自然推广。本次报告将重点汇报黎曼曲面背景下 A 型 Toda 系统有限能量解研究的若干进展：首先探讨穿孔圆盘与穿孔复平面情形下解的精细分类问题；其次详述在紧黎曼曲面框架中，通过非退化亚纯一形式组与锥球度量构造新型解族的两类方法；最后分析此类曲面上解存在的普吕克型必要条件。文中所述成果是与牟景宇、史毅茜、孙天阳及魏春晖合作研究的阶段性工作，其中部分结论尚待进一步完善。

Picard groups of higher real K-theories

段志鹏
南京师范大学

Higher real K-theories, as the homotopy fixed point of Lubin-Tate theories, capture significant periodic information about the stable homotopy groups of spheres. These theories also play an important role in both topology and geometry. In this talk, we explore the Picard groups of higher real K-theories, offering two distinct approaches to studying these groups at general heights: the equivariant approach and the algebraic approach. Additionally, we provide a complete answer to the Picard group of $E_4^{hC_4}$, which serves as a counterexample to the conjecture that all Picard groups of this type arise from suspensions by equivariant spheres.

Shortest filling geodesics on hyperbolic surfaces

王中子
北京大学

The minimal length of filling geodesics on genus g hyperbolic surfaces equals the half of the perimeter of the right-angled $(8g-4)$-gon. This minimum can be realized by a single geodesic. This is a joint work with Yue Gao and Jiajun Wang.

Homological dimension of discrete subgroups in higher rank Lie groups

汪湜
上海科技大学

Given a discrete subgroup H in a higher rank non-compact simple real Lie group G. We show that either H is a lattice in G, or the homological dimension of H is bounded above by (n-1/8r), where n is the dimension of the symmetric space G/K and r is the real rank of G. The proof uses a geometric gradient flow motivated by the Patterson-Sullivan theory and the barycenter map of Besson-

Courtois-Gallot. This is joint work with Chris Connell and Ben McReynolds.

Reverse Lagrangian surgeries

潘宇
天津大学

A major theme in symplectic and contact topology is the study of Legendrian knots and exact Lagrangian surfaces. In the talk, we will talk about some flexibility results of immersed Lagrangian surfaces using augmentation, a Floer type invariant of Legendrian knots. In particular, for an immersed filling of a topological knot, one can do surgery to resolve a double point with the price of increasing the surface genus by 1. In the Lagrangian analog, one can do Lagrangian surgery on immersed Lagrangian fillings to treat a double point by a genus. In this talk, we will explore the possibility of reversing the Lagrangian surgery, i.e., compressing a genus into a double point. It turns out that not all Lagrangian surgery is reversible.

Harmonic map rays and Jenkin-Serrin problem

潘会平
华南理工大学

In this talk, we will discuss several types of ray structures on the Teichmuller space, and show a transition between Teichmuller geodesics and Thurston geodesics via harmonic map (dual) rays. The key of the proof is the solution of a generalized Jenkin-Serrin problem: existence and uniqueness of some tree-valued minimal graphs over hyperbolic domains. This is a joint work with Michael Wolf.

C^0-closedness of $Symp_0(X)$

吴惟为
浙江大学

The C^0 topology of the symplectomorphism groups has lots of mysterious basic questions. The famous symplectic rigidity theorem says that, given any symplectic manifold X, the symplectomorphism group Symp(X) are closed in Diff(X) with respect the the C^0 topology. It is also easy to check that $Symp_h(X)$, the group of homologically trivial symplectomorphisms, is closed in Symp(X). The relation between $Symp_h(X)$ and $Symp_0(X)$ is more delicate.

In this talk, we will present a proof of the closedness of $Symp_0(X)$ in $Symp_h(X)$ when X is a log Calabi-Yau surface of type D, in the sense of Li-Li-Wu. For these symplectic manifolds, it was previously known that $Symp_h(X)$ is a subset of $Diff_0(X)$. Our result implies that, there is a smooth isotopy of some symplectomorphism which cannot be C^0-approximated by a path of symplectomorphisms. This is an ongoing project with Marcelo Atallah and Cheuk-Yu Mak.

Free circle actions on highly connected (2n+1)-manifolds

江怡
首都师范大学

The problem we concern in this talk is to determine which highly connected (2n+1)-manifolds admit free circle actions. I will introduce some previous work and our progress on this problem. This is a joint work with Yang Su.

Spectral gaps on thick part of moduli spaces

张皓皓
清华大学

We study spectral gaps of closed hyperbolic surfaces for large genus. We show that for any fixed $k \geq 1$, as the genus goes to infinity, the maximum of $\lambda_k - \lambda_{k-1}$ over any thick part of the moduli space of closed Riemann surfaces approaches the limit $\frac{1}{4}$. This is a joint work with Yunhui Wu.

Chern Classes of the conjugation representations and invariant polynomials

古星
西湖大学

The theory of characteristic classes of the projective unitary group $PU(n)$ is fundamental yet very mysterious as of today. We will review recent progresses on the study on this topic, and consider the Chern classes of the conjugation representation of the projective unitary group $PU(p^l)$, where p is an odd prime.

Nearly geodesic surfaces are filling

韩肖垄
上海数学与交叉学科研究院

A surface S in a manifold M is filling if S cuts M into contractable components. We prove for any closed hyperbolic 3-manifold M , there exists a K">0 such that every homotopy class of K-quasi-Fuchsian surfaces with 1<K ≤ K" is filling. As a corollary, the set of embedded surfaces in M satisfies a dichotomy: it consists of at most finitely many totally geodesic surfaces and surfaces with a quasi-Fuchsian constant lower bound K". Each of these nearly geodesic surfaces separates any pair of distinct points at the sphere of infinity. Crucial tools include the rigidity results of Mozes-Shah, Ratner, and Shah. This work is inspired by a question of Wu and Xue whether random geodesics on random hyperbolic surfaces are filling.

Kauffman bracket skein module of small Seifert manifolds

史尚君
华东师范大学

We compute the Kauffman bracket skein modules (KBSM) of small Serfeit manifolds by providing presentations of them. From the presentations of small Serfeit manifolds, we show that the KBSM of $D^2(k1, k2)$, $ki \geq 1$ are infinitely generated free modules and the KBSM of $S^2(k1, k2, k3)$, $ki \geq 2$ are finitely generated.

This is joint work with Xiao Wang and Minyi Liang.

The symmetry of Heegaard splittings

陈皓
华东师范大学

The mapping class group of a Heegaard splitting for a 3-manifold is defined as the group of isotopy classes of orientation-preserving diffeomorphisms of the manifold that preserve both handlebodies of the splitting setwise. When viewed as a subgroup of the mapping class group of the Heegaard surface, this object raises fundamental questions posed by Minsky: Is it finite? Is it finitely generated? Is it finitely presented? In this talk, I will discuss recent progress on this problem and present a new approach to address these questions. The talk is based on joint work with Yanqing Zou.

A note on the existence of solutions to Hitchin's self-duality equations

王朔
中国科学技术大学

In 1987, Hitchin introduced the self-duality equations on rank-2 complex vector bundles over compact Riemann surfaces with genus greater than one as a reduction of the Yang-Mills equation and established the existence of solutions to these equations starting from a Higgs stable bundle. In this paper, we fill in some technical details in Hitchin's original proof by the following three steps. First, we reduce the existence of a solution of class L_2^1 to minimizing the energy functional within a Higgs stable orbit of the L_2^2 complex gauge group action. Second, using this transformation, we obtain a solution of class L_2^1 in this orbit. These two steps primarily follow Hitchin's original approach. Finally, using the Coulomb gauge, we construct a smooth solution by applying an L_2^2 unitary gauge transformation to the L_2^1 solution constructed previously. This last step provides additional technical details to Hitchin's original proof.

Counting Rational Curves and Standard Complex Structures on HyperKahler ALEs

吕源玖
中国科学技术大学

All hyperKahler ALE 4-manifolds with a given non-trivial finite group Γ in SU(2) at infinity are parameterized by an open dense subset of a real linear space of dimension 3rankΦ. Here, Φ denotes the root system associated with Γ via the McKay correspondence. Such manifolds are diffeomorphic to the minimal resolution of a Kleinian singularity. By using the period map of the twistor space, we specify those points in the parameter space at which the hyperKahlerian family of complex structures includes the complex structure of the minimal resolution. Furthermore, we count the rational curves lying on each hyperKahler ALE 4-manifold. For each point in the parameter space, we can assign an integer equals to the number of complex structures which contains rational curves. We show this integer function on the parameter space is lower semi-continuous. In the end, we prove that the twistor space of any hyperKahler ALE cannot be Kahlerian.

On the Kelly Monoidal Structure of Λ-sequences

范骜文
中国科学院

We review the Kelly's composition product of Λ-sequences on a symmetric monoidal category. We examine the associativity of the composition product and give the normal op lax monoidal structure on the category of Λ-sequences through monoidal localization. In addition, we construct the correspondence between operads on a general symmetric monoidal category and monoids in the category of Λ-sequences, which will generalize Kelly's and May-Zhang-Zou's correspondence theorem.

Free circle actions on certain simply connected 7−manifolds

胥夫鹏
清华大学

We determine for which nonnegative integers k,\ l and for which homotopy 7−sphere Σ the manifold $\left(kS^2 \times S^5\right) \# \left(lS^3 \times S^4\right) \#\Sigma$ admits a free smooth circle action.